THE EFFECT OF GROUND-MOTION CHARACTERISTICS AND INTENSITY MEASURES ON THE SLIDING OF RIGID BODIES

C. G. Lachanas[1], D. Vamvatsikos[2], M Causse[3] & S. R. Kotha[3]

[1] *School of Civil Engineering, National Technical University of Athens, 15773 Zografou, Athens, Greece,* lahanasch@central.ntua.gr

[2] *School of Civil Engineering, National Technical University of Athens, 15773 Zografou, Athens, Greece*

[3] *ISTerre, Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS, IRD, Univ. Gustave Eiffel, 38000 Grenoble, France*

Abstract: *The sliding response of rigid bodies is investigated under multiple suites of ground-motion records having different inherent characteristics: Ordinary (no-pulse-like, no-long-duration), near field, pulse-like versus spectrally-matched non-pulse-like twins, and long-duration versus spectrally-matched short-duration twins. A basic Coulomb friction model of a rigid block resting freely on a flat surface is used as a testbed, applying incremental dynamic analysis to assess response statistics under the different suites at multiple levels of intensity. Alternative intensity measures are employed, including the peak ground acceleration, the peak ground velocity, and variants of average spectral acceleration—defined as the geometric mean of spectral accelerations over a range of periods. As engineering demand parameters, both the maximum absolute displacement and the absolute residual displacement are employed. The results indicate a non-trivial sensitivity to duration and pulsiveness, and suggest as well that some intensity measures perform considerably better than others in suppressing sensitivity to such peculiar ground-motion characteristics*.

1. Introduction

The seismic response of some structures and/or nonstructural building contents is represented by the theory of the rigid block. In other words, many structures/contents consist of rigid bodies that stand freely on a rigid support base earth/floor (e.g., ancient monuments, bridge piers, server racks, museum artefacts etc.), with their main seismic response mechanisms being *rocking* and/or *sliding*. Rocking and sliding are also proposed as effective response mechanisms for the development of seismic isolation solutions (e.g., Tsopelas *et al.* 1996; Agalianos *et al.* 2017). Figure 1 presents the general case of a rigid block subject to seismic excitation at its base (ground/floor level). For this general case of a planar block model of rectangular shape, the two crucial parameters that govern the type of the seismic response are the slenderness angle $\alpha = \tan^{-1}(2b/2h)$ and the coefficient of friction, μ, between the block and its supporting surface. In cases where $\tan\alpha < \mu$ the block uplifts and initiates rocking between its pivot points O-O' when the excitation's acceleration exceeds the limit of rocking uplift $\mathrm{g}\tan\alpha$ (Housner 1963). At this case the crucial response parameter is the rocking angle θ, which ranges between [0-1) for "safe" rocking response with $\theta \approx 1.0$ signifying the overturning of the block. On the other hand, when $\tan\alpha > \mu$, sliding is the seismic response mechanism for the rigid block; the sliding initiation threshold is the point where the base acceleration exceeds the value of μ. For sliding, the critical response value is the relative displacement of the block with respect to its supporting base, d_{rel}. Theoretically, no collapse level is pre-defined for sliding based on d_{rel} values during seismic response, as the block has simply moved to a different position while sustaining no damage (in contrast, e.g., to an overturned block). Still, practically, the pure sliding response is usually restricted by nearby structures/contents, or other limitations to the available space to accommodate sliding. This α versus μ "competition" for defining if a rigid block will rock or slide on its supporting base means that for a specific interface (constant μ) rocking applies to the slender (low α) blocks whereas sliding characterizes the stocky ones.

The wide application of rocking and sliding in engineering structures has led to significant research effort in both fields. Regarding rocking, a lot of analytical studies exist in the field (e.g., Housner 1963; Makris and Konstantinidis 2003; Dimitrakopoulos and DeJong 2012) investigating in detail the rocking oscillator. More recently Lachanas (2022) has employed a seismic response standardization of the rocking response of simple rocking blocks for application in rocking vulnerability studies. This standardization was effected via a probabilistic treatment of the seismic response and by using state-of-the-art statistical tools to seek answers to important response standardization issues: how many analyses, which statistical distribution fits rocking fragilities, which are the optimal intensity measures, and whether the vertical component of the ground motion is important. Concrete answers may not exist within a one-by-one comparison basis (one block, one ground motion) but they may exist on a population-by-population comparison basis (many blocks, many ground motions). The final step of this standardization was the development of closed-form equations for rocking fragilities that can be employed both by practitioners and researchers for the rapid seismic design and/or assessment of rocking structures located on-ground or on the higher floor of buildings (Lachanas 2022; Kazantzi *et al*. 2021; Kazantzi *et al*. 2022).

In a similar pattern, considerable research effort exists in the field of sliding block response including both analytical and experimental studies (e.g., Newmark 1965; Choi and Tung 2002; Konstantinidis and Makris 2005, 2009; Konstantinidis and Nikfar 2015; Nikfar and Konstantinidis 2017). Still, considering the probabilistic treatment of the sliding response within the performance-based earthquake engineering framework (Cornell *et al.* 2002) there is still a need for some answers before one proceeds to the construction of closed-form equations for sliding fragilities. As a first step towards this desired standardization of the sliding response of simple rigid bodies, we examine herein the influence of the inherent characteristics of the ground motion characteristics (e.g., *pulsiveness*, duration etc.) on the sliding response of rigid blocks. A comparative study is presented testing the sliding response of rigid blocks under multiple suites of ground motions with different characteristics. Moreover, the use of alternative intensity measures (IMs) is investigated as a potential way to reduce or hopefully eliminate the differences in the seismic response stemming from inherent characteristics of the ground motion.

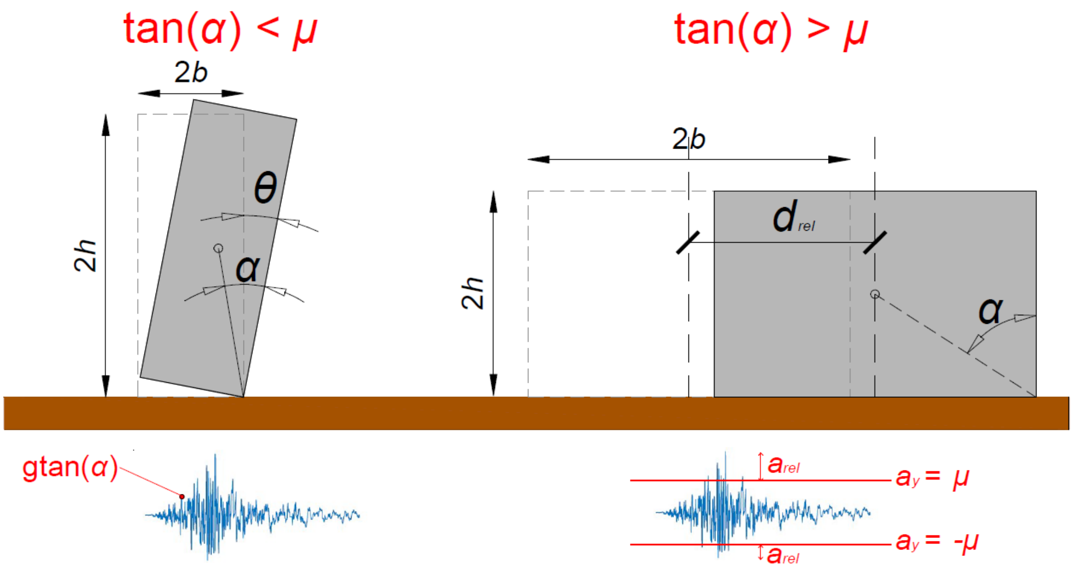

Figure 1. Rigid block on a rigid base subjected to seismic excitation. Rocking response (left) and sliding response (right) of the block.

2. Modeling and analysis choices

For the analysis, the planar rigid sliding block model of Figure 1 (right) was employed for the case of two-way sliding response, using the software implementation of Tsarpalis *et al*. (2022). This is based on Newmark's sliding block analysis (Newmark 1965) and it has been validated as an efficient model for assessing the response of sliding rigid bodies against "flat slider" finite element models. Its only parameter is the static coefficient of friction μ, assumed to be velocity and pressure independent (Coulomb 1776). A constant value of $\mu = 0.45$ was assumed for the examples presented herein. Based on Figure 1 and given the model at hand, rigid bodies with $\tan\alpha > 0.45\text{rad}$ are prone to slide when the ground acceleration exceeds the value of 0.45g.

Incremental dynamic analysis (IDA, Vamvatsikos and Cornell 2002) was employed as the tool for analysing the seismic response and estimating the corresponding response statistics. One horizontal component of the ground motion was assigned to the planar model at hand for the calculation of the relative displacement of the block. Figure 2 presents the IDA curves for a set of 105 ordinary (no-pulse-like, no-long-duration) ground motions (Lachanas 2022) when using the peak ground acceleration (PGA) as IM and the absolute residual relative displacement of the block (d_{res}) or its absolute maximum relative displacement (d_{max}) as engineering demand parameters (EDPs). For the analysis, a constant step of 0.01g was employed for scaling the PGA of the ground motions, whereas a considerably high upper limit of 4g was set for stopping the scaling procedure since no "collapse" level is predefined for the generic case of a free-to-slide rigid block. Moreover the 16/50/84% EDP given IM (EDP|IM) fractiles are presented in Figure 2. Therein, sliding initiates when $PGA > \mu$, whereas high record-to-record dispersion is observed in general for the PGA-based IDAs when moving away from the sliding initiation. In addition, sliding IDAs of PGA versus d_{res} (Figure 2a) show higher variability than those of PGA versus d_{max} (Figure 2b). This is consistent with similar observations on the residual versus peak response of yielding oscillators and buildings (Ruiz-Garcia and Miranda 2006, 2010).

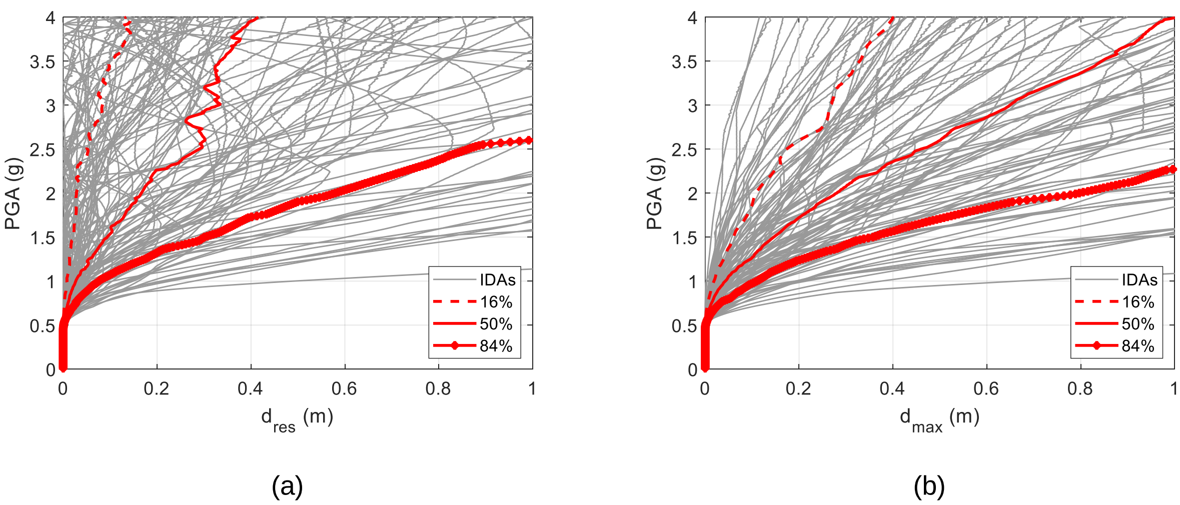

Figure 2. IDA curves under a suite of 105 ordinary ground motions with the corresponding 16/50/84% EPD|IM quantiles for PGA as IM versus (a) d_{res} as EDP and (b) d_{max} as EDP.

3. Comparing the seismic response via multiple suites of ground motions

The influence of the inherent characteristics of the ground motion on the seismic response of sliding rigid blocks is investigated by comparing the 16/50/84% IDA quantiles using seven different sets of ground motions:

1. A set of 105 ordinary ground motions (Lachanas 2022) having magnitude $M_w > 6.2$ and $PGA > 0.14\text{g}$ (Figure 2)
2. An alternate set of 115 ordinary ground motions, none of which belongs to set #1 while still fulfilling the exact same criteria
3. A set of 192 pulse-like ground motions (Kohrangi *et al.* 2019)
4. A set of 192 non-pulse-like ground motions selected to be spectrally equivalent on one-to-one basis with the motions of set #3 (Kohrangi *et al.* 2019)
5. A set of 146 long duration ground motions (Chandramohan *et al.* 2016)

6. A set of 146 short duration ground motions selected to be spectrally equivalent on one-to-one basis with the motions of set #5 (Chandramohan *et al.* 2016)
7. A set of 128 near-field ground motions, of which almost half (67) are pulse-like ground motions whereas the rest (61) are non-pulse-like (NESS2, Sgobba *et al.* 2021)

Figure 3 presents the 16/50/84% EDP|IM IDA quantiles per suite of ground motions employed, when using PGA as IM and d_{res} (Figure 3a) or d_{max} (Figure 3b) as EDP. A single color has been used to denote each suite; a dashed line is used to plot the 16% EDP|IM quantile, a solid for the 50%, and a solid-plus-stars for the 84%. As illustrated, considerable an intricate pattern of differences and similarities are captured between the suites of ground motions for both EDP cases.

First of all, it is worth mentioning that no differences are captured in the IDA quantiles between the two sets of 105 and 115 ordinary ground motions (Sets #1 and #2). This means that such a high number of ground motions is adequate for assuring the fidelity of the corresponding response statistics. Furthermore, any differences between sets appear after the sliding initiation neighborhood $[d_{max}, d_{res}] > 1 - 2\text{cm}$, since the sliding initiation threshold in all cases is directly associated with the ground acceleration, as shown in Figure 1. Thus, similarly with rocking (Lachanas *et al.* 2023) PGA is the most efficient sliding IM close to the sliding initiation. It becomes highly inefficient for the non-zero EDP range of response capturing high record-to-record dispersion for both d_{res} and d_{max}.

Now, when comparing Sets #1 and #2 with Set #3, *pulsiveness* is found to have a strong impact on the sliding displacement of rigid blocks, at least when PGA is employed as IM, consistently leading to higher EDP demands. However, the spectrally matched twins of pulse-like versus no-pulse-like ground motions (Sets #3 and #4) show no such differences in any of the three quantiles, widely differing from Sets #1 and #2 of “unmatched” ordinary ground motions. In other words, *pulsiveness* is important, but it seems that its effect can be fully captured by accounting for the spectral shape. This leaves an opening for potentially removing the effect of *pulsiveness* by employing an IM that better captures the spectrum at periods longer than 0s.

The duration of the ground motion also affects the peak or the residual sliding response mainly in high intensity levels; this is where the IDA quantiles of the long-duration ground motions set (Set # 5) differs from those of the ordinary Sets #1 and #2. In this case, matching of the spectral shape does not seem to work as efficiently as in the case of the pulse-like ground motions to remove the differences observed. Considerable mismatch is still being captured in high intensities between the spectrally-matched Sets #5 and #6 of long and short duration ground motions, respectively.

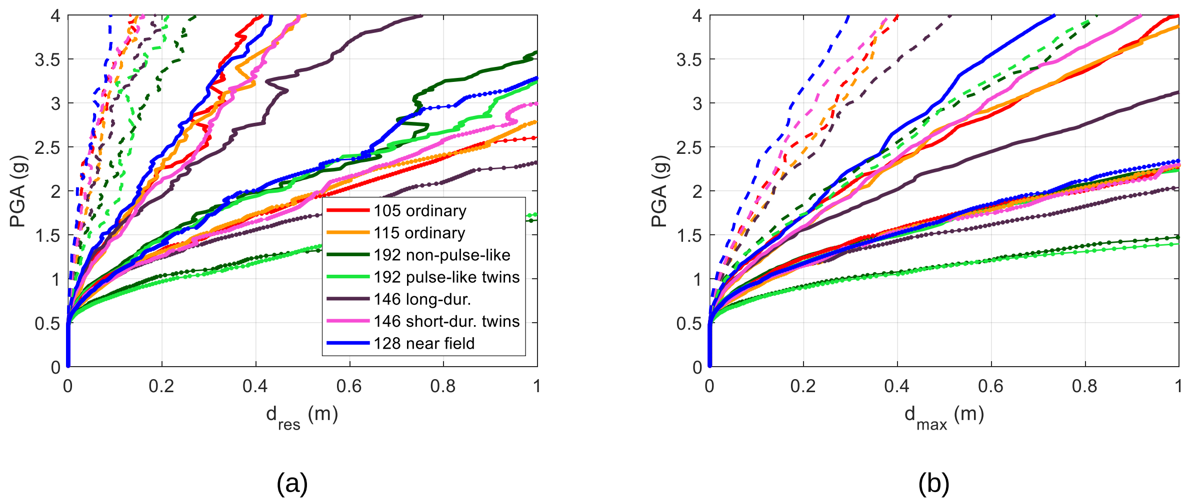

Figure 3. 16/50/84% EDP|IM IDA quantiles under multiple suites of ground motions for PGA as IM versus (a) d_{res} as EDP and (b) d_{max} as EDP. 16% quantiles are shown in dashed lines, 50% in solid lines and 84% in solid-plus-stars lines.

Regarding the NESS2 Set #7, moderate differences are captured in the IDA quantiles with respect to Sets #1 and #2 of ordinary ground motions. Specifically, the 16% and 50% EDP|IM demands for the NESS2 set are somewhat lower than those of the ordinary sets in terms of the peak sliding displacement. Still, given the overall large dispersions, it is not entirely obvious whether this difference is statistically significant. Either way, its magnitude is such that one can reasonably claim that the inherent characteristics of mixed (pulse-like, non-pulse-like) near-field sets do not have a strong impact on the sliding response when compared with solely ordinary or solely pulse-like sets. The implications of this observation extent to the assessment of hazard and the selection of ground motion records in the near field in a manner similar to Tarbali *et al.* (2019).

4. Examine alternative IMs

Motivated by our findings with regard to pulsiveness we now turn to alternative IMs. Figures 4, 5 present the EDP|IM IDA fractiles of the seven suites of ground motions when using the peak ground velocity (PGV) and different cases of the average spectral acceleration ($AvgS_a$) as IMs versus d_{res} and d_{max}, respectively. $AvgS_a$ is defined as the geometric mean of the elastic spectral accelerations over a period range (Cordova *et al.* 2001; Vamvatsikos and Cornell 2005). Two different cases of period ranges are examined herein with a constant step of $0.1s$ in order to investigate different areas of the elastic spectra. Specifically, a broad range of medium-to-high periods that range within $0.5 - 4.0s$ is employed for $AvgS_{a1}$. On the other hand, a narrower range of periods between $1.0 - 2.5s$ is applied for $AvgS_{a2}$.

Figure 4 shows the 16/50/84% EDP|IM quantiles per suite of ground motions for the case of PGV. As observed, the differences between the quantiles of the different suites of ground motions are in general lower than those captured in Figure 3 for PGA. Specifically, the influence of the *pulsiveness* seems to be considerably reduced in comparison with PGA, especially for the residual sliding displacement. Still, the pulse vs ordinary twins (Sets #3 and #4) seem to differ from the other sets but now in a different trend, producing lower seismic demands for d_{res} and d_{max}, rather than higher ones. Still, whether this difference is statistically significant remains a question. Regarding the duration of the ground motion, it still shows some influence leading to higher seismic demands especially for the residual sliding response in high PGV levels. Nevertheless, this effect is reduced when compared to the PGA results of Figure 3. For the NESS2 Set #7, the differences that were captured in the median response for the high PGA levels and d_{max} in Figure 3 are also attenuated. Additionally, the dispersion within each individual set is reduced in comparison with PGA, except for the sliding initiation neighbourhood where PGA can be proclaimed as the unbeaten champion of any potential IM. Hence, away from sliding initiation, PGV can be considered as a more efficient sliding IM than PGA.

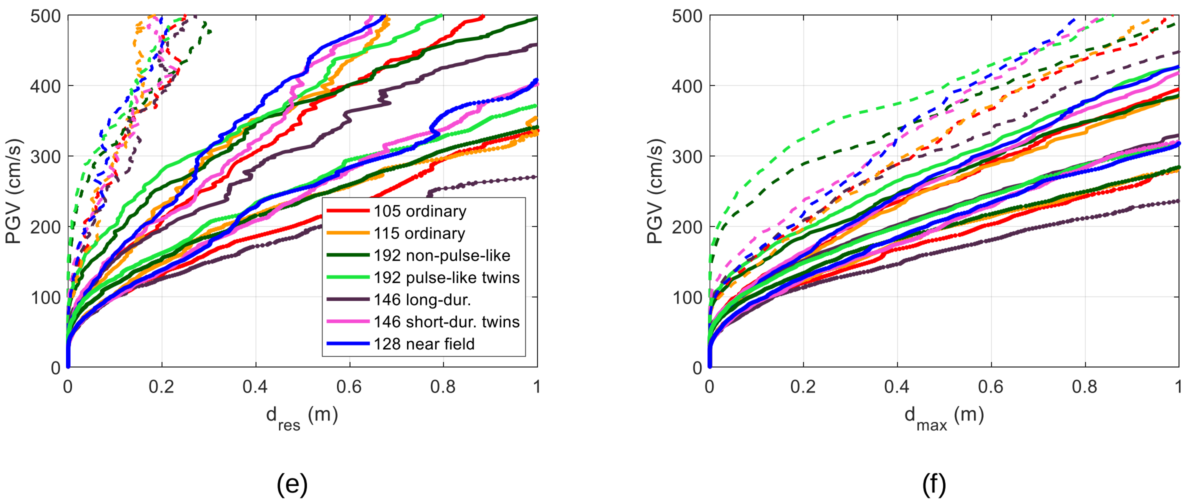

Figure 4. 16/50/84% EDP|IM IDA quantiles under multiple suites of ground motions for PGV, as IM versus (a) d_{res} as EDP and (b) d_{max} as EDP. 16% quantiles are shown in dashed lines, 50% in solid lines and 84% in solid-plus-stars lines.

The findings for the two $AvgS_a$ cases tested are similar or ever better than those of PGV. As shown in Figure 5 for both $AvgS_{a1}$ and $AvgS_{a2}$ the inherent characteristics of the ground motions suites offer lower influence on the sliding response statistics than even PGV. With respect to *pulsiveness* the same (or even better) findings are observed for $AvgS_{a1}$ and $AvgS_{a2}$ as for PGV. It may be claimed here that the effect of spectral matching observed earlier was a herald for this result, as $AvgS_a$ and PGV are better average indicators of the overall spectrum than PGA. In terms of the signal duration, both $AvgS_a$ cases are found to perform better than PGV. Hence, $AvgS_a$ can be considered as the "better" performing sliding IM of the three compared herein since it can almost "hide" the impact of the characteristics of the ground motions especially when d_{res} is employed as EDP. For d_{max}, the *pulsiveness* still shows some influence. Moreover, the record-to-record dispersion is reduced significantly especially for the case of d_{max} in comparison with PGV and PGA. However, $AvgS_a$ "loses" to PGA in the sliding initiation neighbourhood, while it also comes with an inherent main disadvantage. Its performance is deeply affected by the chosen period range, which needs to be properly adjusted. For instance, analyses employed by the authors (not shown herein for brevity) with different cases of $AvgS_a$ have shown that assuming a narrow range of periods from the short-period area of the elastic spectra can lead to less robust $AvgS_a$ choices relative to the ones shown herein.

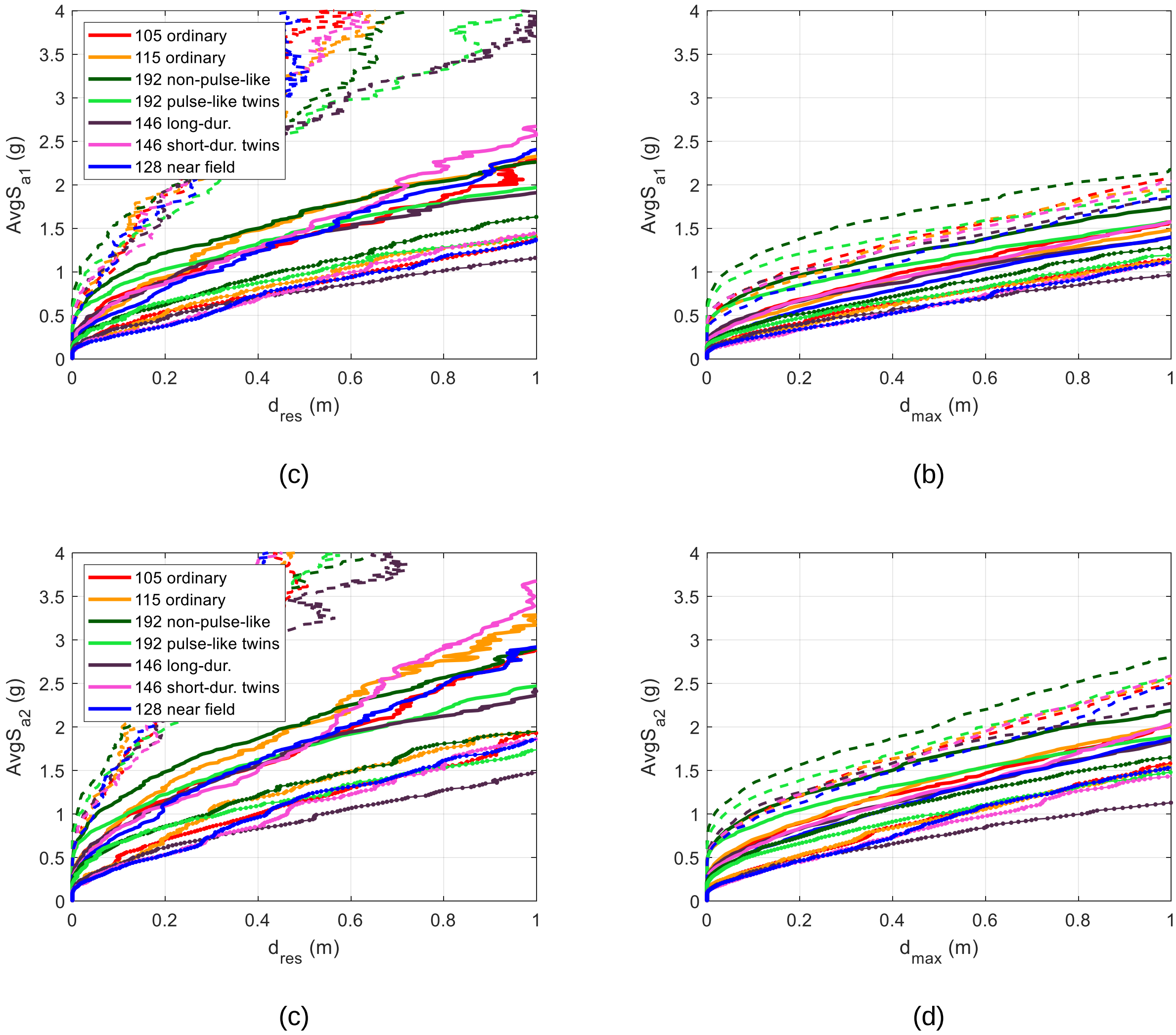

Figure 5. 16/50/84% EDP|IM IDA quantiles under multiple suites of ground motions for $AvgS_{a1}$, and $AvgS_{a2}$ as IMs versus (a, c) d_{res} as EDP and (b, d) d_{max} as EDP. 16% quantiles are shown in dashed lines, 50% in solid lines and 84% in solid-plus-stars lines.

5. Conclusions

Comparing the sliding response statistics of simple rigid blocks via IDA for the 7 suites of ground motions with different characteristics, the main conclusions of the present study are listed below:

1. The inherent characteristics of the ground motion can have a strong impact on the sliding response. Specifically, the use of pulse-type ground motions can lead to consistently higher or lower (depending on the IM) seismic demands in comparison with the usage of solely ordinary ground motions. The use of spectrally matched twin sets of pulse-like versus ordinary ground motions seems to eliminate the differences in the seismic demand calculations. On the other hand, the duration of the ground motion is found to impact the sliding demands mainly in the higher IM levels leading to higher seismic demands. At this case, the matching of the spectral shape is not found to be as efficient as in the pulse-like ground motions case. The use of a mixed (pulse-like, no-pulse-like) suite of near field ground motions is found to have relatively low impact on the seismic demand calculations in comparison with sets of solely ordinary ground motions.
2. The sensitivity to *pulsiveness* and duration is distinctly outlined in the 16/50/84% IDA quantiles when PGA is employed as IM both for residual and maximum absolute displacement as EDPs. Regardless, PGA remains the most efficient IM close to the sliding initiation threshold due to its direct association with the base (ground/floor) acceleration that defines the initiation of sliding.
3. For PGV and different period-range cases of $AvgS_a$, the sensitivity to the inherent characteristics of the ground motion is considerably reduced. For the latter, broad period-range cases from low-to-high periods or moderate-length ranges of medium-to-high periods are better options for sliding IMs in comparison with narrow low-period ranges. Moreover, with the exception of the sliding displacement levels at the neighbourhood of sliding initiation, the record-to-record dispersion for these two IMs is lower than that of PGA. Hence, both can be considered as more efficient IM choices for the case of sliding rigid blocks.

All these findings can be considered as a first step towards the seismic response standardization for simple sliding blocks for risk and vulnerability assessment studies as well as for the development of application-oriented Ground Motion Models.

6. Acknowledgements

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